Monopolistically Skewed Business Cycles

Frederik Bennhoff [1] Timo Haber [2] Niklas Schmitz [3]

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[1]University of Zurich

[2]De Nederlandsche Bank
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[3]Point72, L.P.
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Intro

Topic and Roadmap

This paper studies a **transmission mechanism of aggregate shocks.**

1. **New Stylized Facts:**
   - Observe how the **outcome distribution** of firm growth rates shifts and **changes shape over the business cycle**
   - Establish heterogeneity: **distribution shifts differently for large v. small firms**
2. **New Theory:** Propose model of **transmission mechanism**:

aggregate cost shock $u_t \uparrow$	$\longrightarrow$	cross-sectional distribution of input shocks, $\epsilon_{i,t}$	$\longrightarrow$	skewed c-sec. output distribution of growth rates $\ln \frac{q_{i,t}}{q_{i,t-1}}$

3. **Empirics:** Test model predictions

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heterogeneous exposures

(heterogeneous) market power

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Recap: Skewness

- Skewness measures compare the **thickness of left tail** to **right tail**
- More mass in the left than right tail $\iff$ negative skewness
- **Kelly Skewness:** $\mathrm{skew}[X] = \frac{[X]_{0.1}+[X]_{0.9}-2[X]_{0.5}}{[X]_{0.9}-[X]_{0.1}}$

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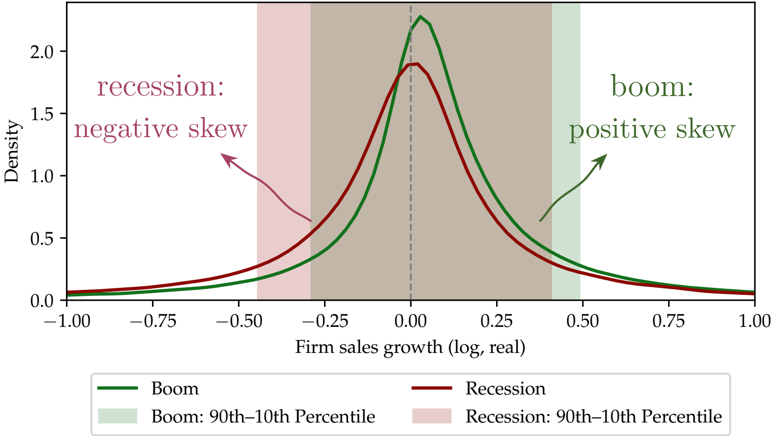

Recap: Literature Business Cycle Moments

1. μ_t mean growth rates are (obviously) pro-cyclical
2. σ_t growth rate and shock variances are counter-cyclical
   - Seminal work on uncertainty as result *and driver* of business cycle (Bloom, 2009)
   - Empirical evidence of negative correlation of μ_t and σ_t (Higson et al., 2002, 2004)
   - Income risk increases in recessions (Guvenen et al., 2014)
   - Aggregate shocks + het. exposures $\implies$ increase in variance (Davis et al., 2025)
3. γ_t skewness is procyclical.
   - Procyclical skew of sales growth rate (Salgado et al., 2025)
   - Procyclical skew of employment growth (Ilut et al., 2018)

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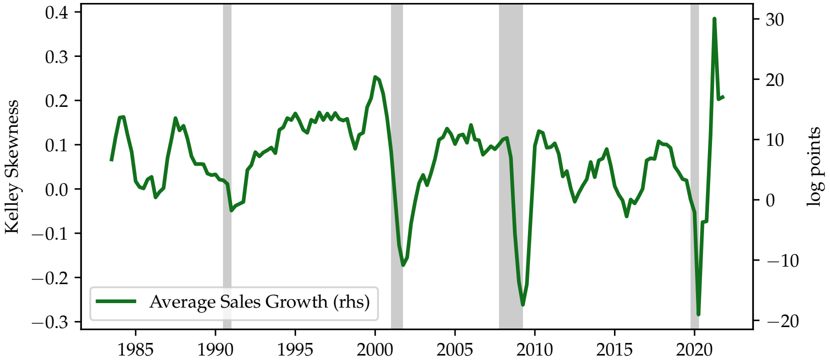
Stylized Facts

Pro-Cyclical Skewness



- Cross-sectional **skewness,** γ_t closely tracks **aggregate real sales growth,** μ_t
- Holds **robustly across time** and skewness measures
- **Split the sample in 'large' and 'small' firms to learn more...**

▸ Robustness: Other skewness measures

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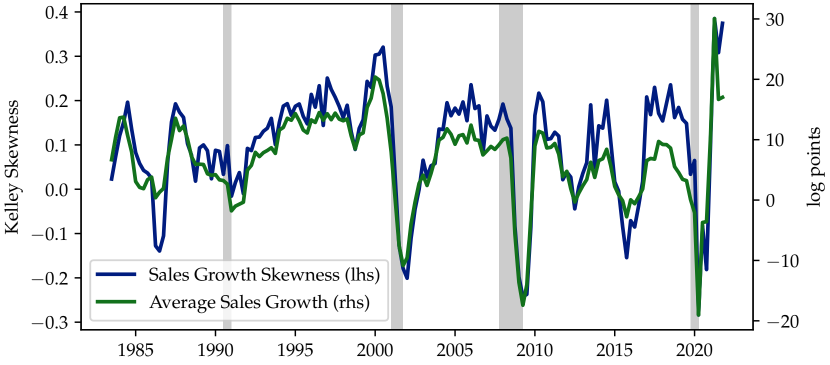

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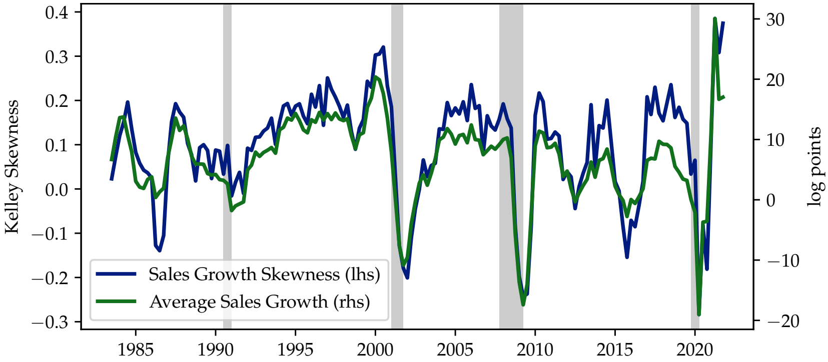

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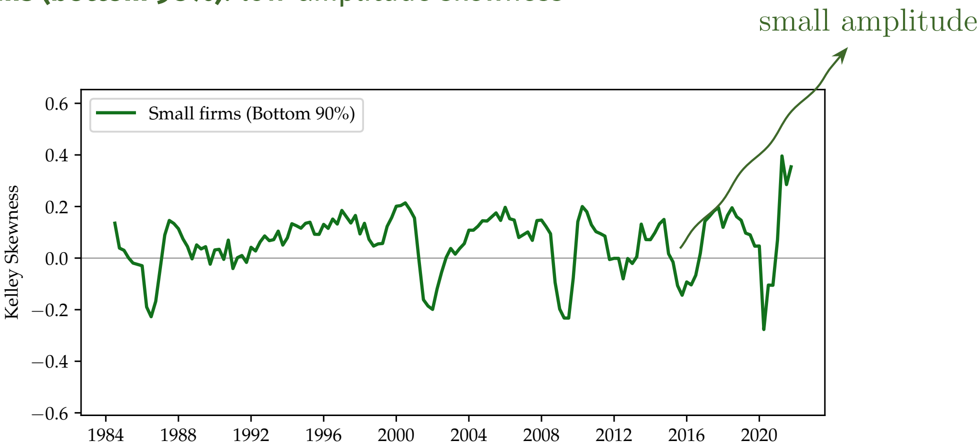
Procyclicality Among Size Groups

Small firms (bottom 90%): low-amplitude skewness

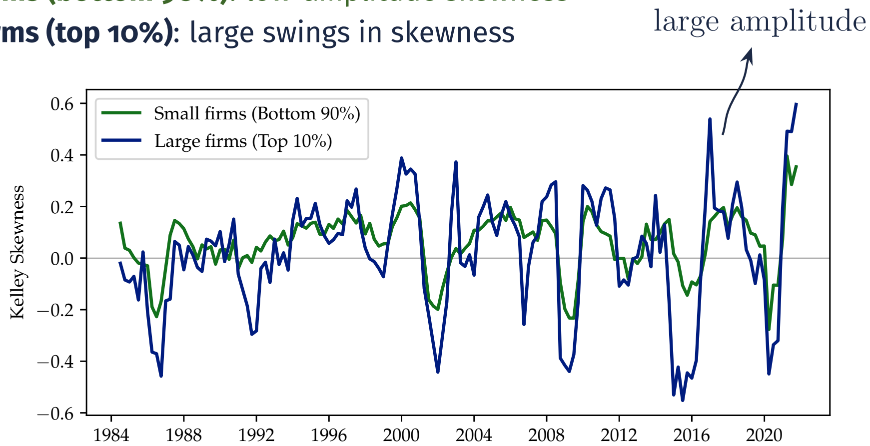


Note: 'Small' refers to the bottom 90% in the Compustat sample; these firms are large on a global scale.

▸ Robustness: Size cutoffs ▸ Robustness: Global Sample ex. U.S.

Procyclicality Among Size Groups

Small firms (bottom 90%): low-amplitude skewness
Large firms (top 10%): large swings in skewness



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Variance and Skewness

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Table 1: Effects of Std. Dev. on Skewness

	All Firms	Top 10%	Bottom 90%
β	-1.137*	-3.281***	-1.788***
	(0.650)	(0.711)	(0.448)
Observations	150	146	146

Note: This table shows the effect of a one-unit increase in the cross-sectional standard deviation of sales growth on skewness of firm-level sales growth. Robust (HAC) standard errors in parentheses. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

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Stylized Facts Summary

We establish following facts empirically:

Regime / Metric	**Skewness**	
	Small Firms	**Large Firms**
Recession	$-$	$--$
Expansion	$+$	$++$
Regression $\Delta\gamma_t$ on $\Delta\sigma_t$	$0 > \beta_{\text{small}} > \beta_{\text{large}}$	

Table 2: Skewness Patterns by Firm Size and Regime

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New: size gradient

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New: relating skewness to variance

Theory

Strategy

1. Show that aggregate shocks (u_t) + idiosyncratic exposures (λ_i) imply heterogeneous input shocks ($\epsilon_{i,t}$) to cost/productivity at the firm level

$\Rightarrow$ Countercyclical variance $\sigma_t^2 \equiv var(\epsilon_{i,t})$, if expansions have $u_t \approx 0$ and recessions have $u_t \ll 0$.

2. Show that heterogeneous input shocks lead to skewed output growth rates which...

a. vary pro-cyclically ($corr(\gamma_t, \mu_t) > 0$) if we have counter-cyclical variance ($corr(\sigma_t, \mu_t) < 0$), and
b. vary with larger amplitude if firms have a higher market power index, α,

given some (realistic) sufficient conditions on inverse demand, $p(q)$.

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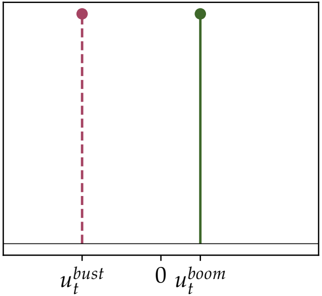
aggregate cost

shock u_t ↑

(a) Aggregate Impulse



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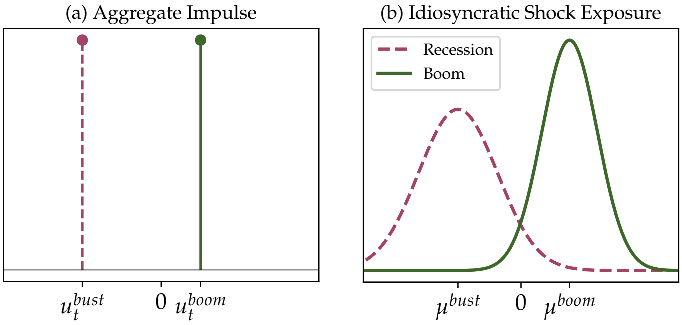
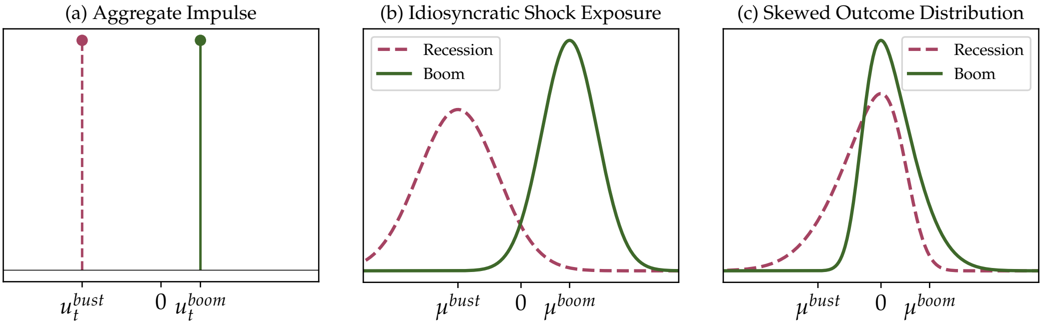


λ_i exposure of firm i:

$\epsilon_{i,t} = \lambda_i u_t$

aggregate cost shock $u_t \uparrow$ $\longrightarrow$ cross-sectional shock distribution, $\epsilon_{i,t}$, with **countercyc. variance** $\longrightarrow$ **procyc. skewed** output dist. of growth rates $\ln \frac{q_{i,t}}{q_{i,t-1}}$



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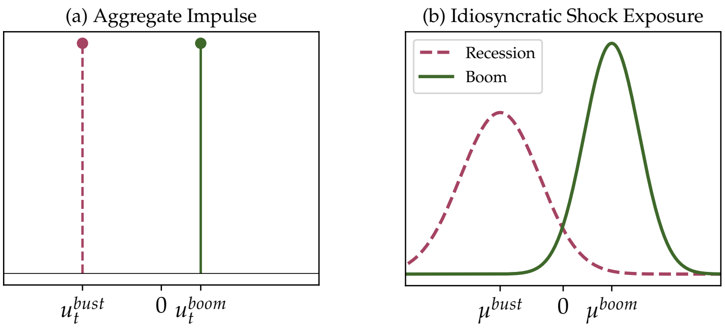
Output policy $Q^{\star}$ is a function of shock $\epsilon_{i,t}$ and market power α:
$\Delta \log q_{i,t} = \Delta Q^{\star}(\epsilon_{i,t}; \alpha)$

1. Aggregate Shocks, Heterogeneous Exposures and Countercyclical Variance



Motivation for Countercyclical Variances

Why would there be a counter-cyclical variance?

- Idea (Davis et al., 2025): Cross sectional variance comes from heterogeneous exposures to aggregate shocks, u_t
- u_t^2 gets large $\implies$ some firms profit, others suffer $\implies$ large variance

Formally...

- Unit measure of firms $i \in [0, 1]$ with shocks $\epsilon_{i,t}$.
- Aggregate risk factors $u_{l,t}$ with $l = 1, ..., L$
- Want to show: $\mathbb{V}(\epsilon_{i,t} \mid \text{recession}) > \mathbb{V}(\epsilon_{i,t} \mid \text{expansion})$

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Shock-Exposure Structure

Suppose, cost-shocks of firms can be written as:

$$\epsilon_{i,t} = e_{i,t} + \sum_{l=1}^{L} \tilde{\lambda}_{i,l} u_{l,t} = e_{i,t} + \boldsymbol{\lambda}_i^T \boldsymbol{u}_t + \bar{\boldsymbol{\lambda}}^T \boldsymbol{u}_t$$

- $e_{i,t}$: i.i.d. idiosyncratic shock.
- $\boldsymbol{u}_t$: Vector of aggregate factors
- $\lambda_{i,l}$: centered shock exposures, unit variance (w.l.o.g.).
- $\bar{\boldsymbol{\lambda}} \leq 0$: exposure w/ negative mean (implies that $u_t < 0$ drives up costs; recession interpretation)

Cross-Sectional Variance

Let factor l become large: $|u_{l,t}| \gg 0$, then:

$$\mathbb{V}_t(\epsilon_{i,t}) = \mathbb{V}(e_{i,t} + \boldsymbol{\lambda}_i^T \boldsymbol{u}_t + \bar{\boldsymbol{\lambda}}^T \boldsymbol{u}_t \mid \boldsymbol{u}_t) \propto u_{l,t}^2$$

- Large aggregate shocks $\Rightarrow$ high cross-sectional variance.
- Assume following pattern:

$$\begin{cases} \text{Recessions} \sim u_t \ll 0\text{: thus variance} \uparrow, \\ \text{Expansions} \sim u_t \approx 0\text{: thus variance is small in normal times.} \end{cases}$$

- Pattern is consistent with u_t following a left-skewed *time-series distribution* which regularly realizes at small values and occasionally in the disaster-tail.
- **Result: countercyclical shock-variance**

Next: How does this pattern drive higher moments?

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- Pattern is consistent with u_t following a left-skewed *time-series distribution* which regularly realizes at small values and occasionally in the disaster-tail.
- **Result: countercyclical shock-variance**

Next: How does this pattern drive higher moments?

Cross-Sectional Variance

Let factor l become large: $|u_{l,t}| \gg 0$, then:

$$\mathbb{V}_t(\epsilon_{i,t}) = \mathbb{V}(e_{i,t} + \boldsymbol{\lambda}_i^T \boldsymbol{u}_t + \bar{\boldsymbol{\lambda}}^T \boldsymbol{u}_t \mid \boldsymbol{u}_t) \propto u_{l,t}^2$$

- Large aggregate shocks $\Rightarrow$ high cross-sectional variance.
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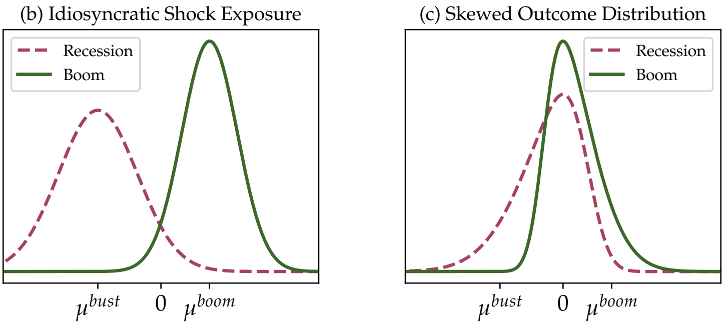
2. Countercyclical Variance, Procyclically Skewed Growth Rates and Market Power



Growth Rates

Let q be output quantity and $\hat{q} \equiv \ln q$. Firm **time series growth rates** are

$$\hat{q}_t - \hat{q}_{t-1}.$$

Static point of departure: First derive conditions under which, cross-sectionally, **log output** is skewed:

$$\text{skew}[\hat{q}] < 0.$$

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General Setup

Consider a monopolistic firm optimizing over output, q.

Cost Function

- Convex costs: $c(q;\epsilon) = q^{\eta} e^{\epsilon}, \quad \eta > 1$
- Stochastic cost shifter: $\epsilon \sim (0, \sigma^2)$, **symmetric**, zero-mean, finite variance

(alternative with linear cost also works)

Demand

- General **inverse demand**: $p(q)$,
- Local regularity assumptions: strictly decreasing, log-concave, thrice differentiable
- Firm is **price taker** if $p(q) = \bar{p} = const.$

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Price Taker's Problem

Optimization Problem

$$\max_{q \geq 0} q\bar{p} - c(q;\epsilon)$$

First Order Condition

$$c'(q_{\text{pt}};\epsilon) = \bar{p},$$

which implies marginal cost pricing.

Equilibrium log-output ($\widehat{q_{\text{pt}}}$) is given by

$$\widehat{q_{\text{pt}}} = \frac{\ln \bar{p} - \ln \eta - \epsilon}{\eta - 1}$$

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where $(\mathcal{E}f)(x) \equiv \frac{f'(x)}{f(x)}x$ is the **elasticity operator** and mr is **marginal revenue**.

Equilibrium Markup is given by Lerner-condition:

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The solution, of the monopolist's problem is unique, interior (positive) and implicitly given as the solution to the first-order condition equating marginal cost to marginal revenue:

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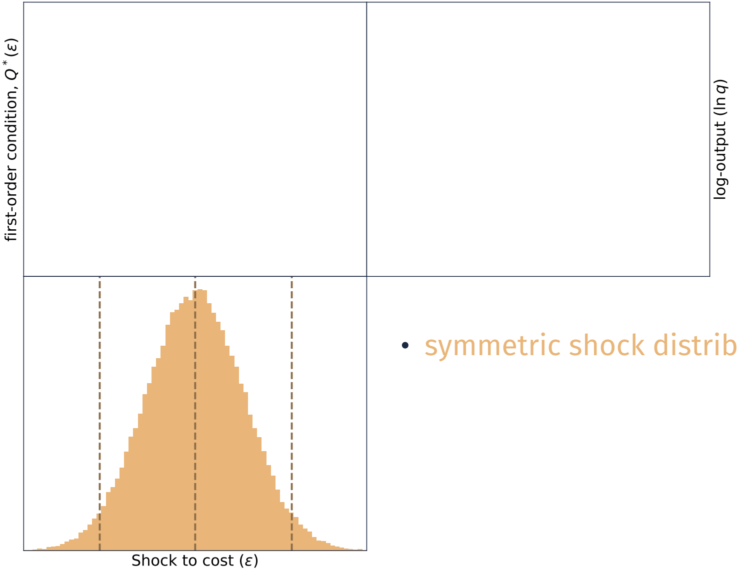

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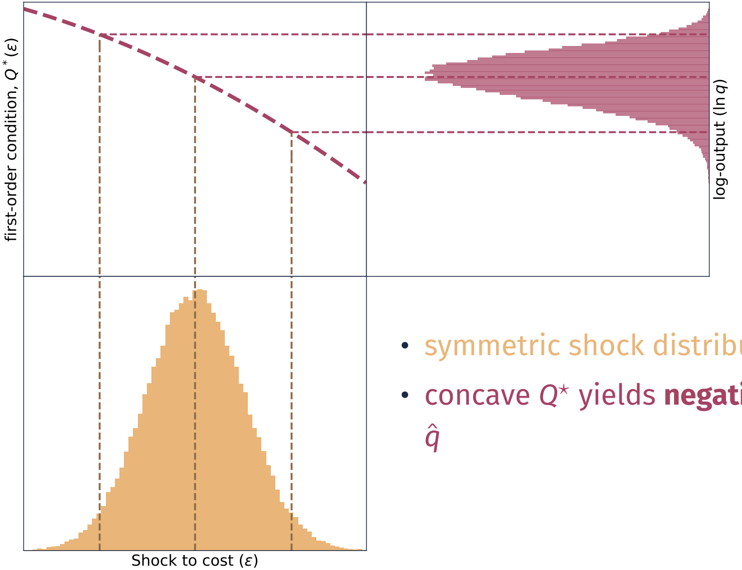

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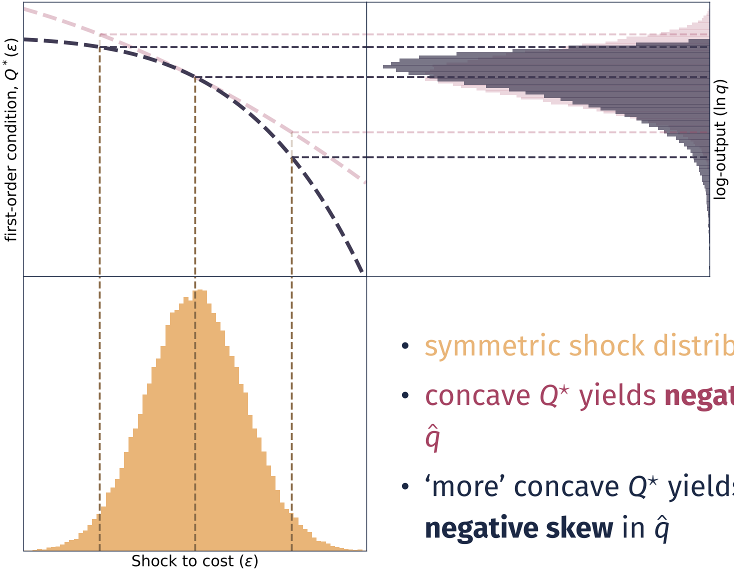

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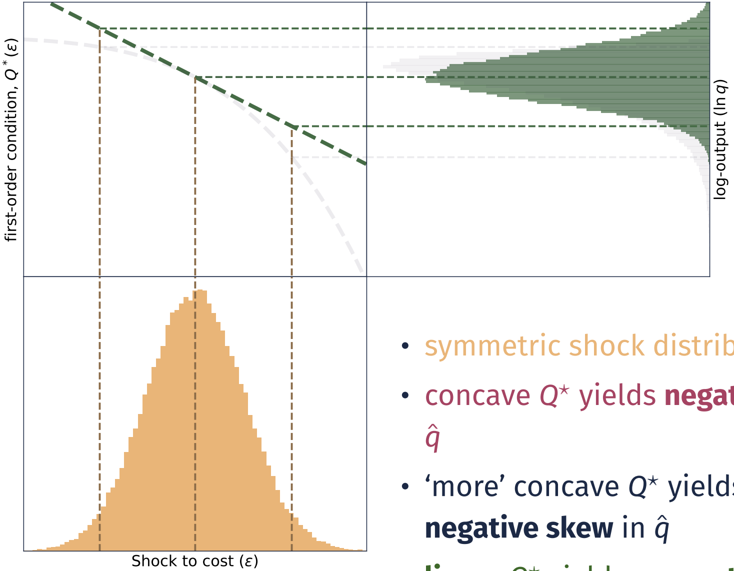

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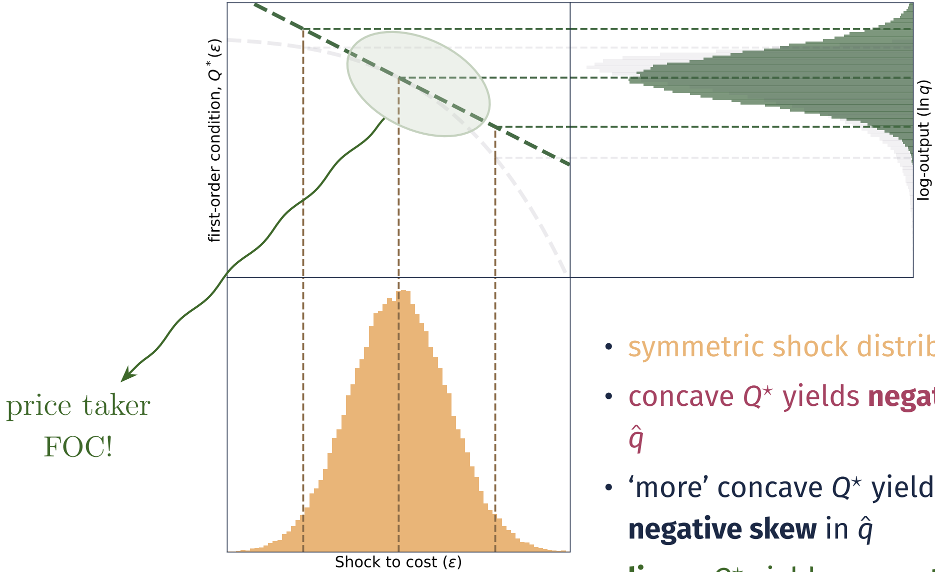

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Because the $Q^{\star}$ is linear for the price taker:

Result: Price Taker

Log output $\hat{q}_{pt}$ and **time-series growth rates** $\hat{q}_{\text{pt},t} - \hat{q}_{\text{pt},t-1}$ of the price taking firm are **unskewed**:

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Examine $Q^\star$ with properties of inverse demand, p

Property 1: MSLD

We say that **Marshall's Second Law of Demand** (MSLD) holds if for all $q \in D$, the elasticity of inverse demand is increasing: $|\frac{\partial}{\partial q}\mathcal{E}p(q)| > 0$.

$\Leftrightarrow$ **The absolute price elasticity of demand increases as the price rises.**

- Interpretation: Consumers become increasingly sensitive to price changes as goods become more expensive.
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Monopolist's output is more left-skewed output than price taker's under MSLD': Let $\hat{q} = Q^*(\epsilon)$ be log-output of a monopolistic firm, $\hat{q}_{\mathrm{pt}}$ be that of the price-taker, and suppose *MSLD*' holds strictly. Then,

$$\mathrm{skew}[\hat{q}] < \mathrm{skew}[\hat{q}_{\mathrm{pt}}] = 0.$$

Nice, if we assume that 'small' firms are all price takers; but we would like to differentiate firms better.

Next: **Introduce market power parameter** $\alpha \in [0, 1]$ to make binary comparison (monopolist ($\alpha = 1$) v. price taker ($\alpha = 0$)) continuous!

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With market power parametrized by $\alpha \in [0, 1]$ (firm with market power α has output $\hat{q}_\alpha$), can we generate **Monotone Skewness**?

Property 3: Monotone Skewness

We say **Monotone Skewness** holds if the skewness index is decreasing in market power. That is, $\mathrm{skew}[\hat{q}_\alpha] \leq 0$ is decreasing in α, with $\mathrm{skew}[\hat{q}_1]$ equaling monopolist and $\mathrm{skew}[\hat{q}_0] = 0$ equaling price-taker output, respectively.

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Pass-Through Rates

The **pass-through**, τ, is defined as the share of a cost increase that is passed on to customers in equilibrium.

Formally, τ equals one plus the elasticity of the mark-up with respect to the cost shifter $\bar{c} \equiv e^{\epsilon}$.

Property 4: IPT

Let the **pass-through** be the share of a cost increase that is passed on to customers in equilibrium given by $\tau(\bar{c}) = 1 + \frac{d \log \mu}{d \log \bar{c}}$. An inverse demand function p features **increasing pass-through (IPT)** if $\frac{\partial}{\partial \bar{c}} \tau(\bar{c}) \geq 0$.

Interpretation IPT: higher-cost firms pass on larger shares of cost increases, lower cost firms absorb some by dropping mark-ups.

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Relationship of IPT and Monotone Skewness

Result: Sufficient Conditions for Monotone Skewness

Increasing pass-through rates (*IPT*) and Marshall's second law of demand (*MSLD*) are sufficient conditions to guarantee that skewness of log-output is negative and decreasing in market power:

$$IPT \wedge MSLD \implies MSLD' \qquad \text{and}$$
$$IPT \wedge MSLD \implies \text{Monotone Skewness}$$

Recap & Outlook

- We thus have a theory which predicts that log-output of larger firms is more left-skewed than of smaller firms.
- **What does it say about skewness of time-series growth rates?**
- **Does it imply pro-cyclically skewed growth rates?**

To this end, we take a look at the role of cross-sectional shock variance, σ^2.

Properties in the literature

Recap & Outlook

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- **What does it say about skewness of time-series growth rates?**
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To this end, we take a look at the role of cross-sectional shock variance, σ^2.

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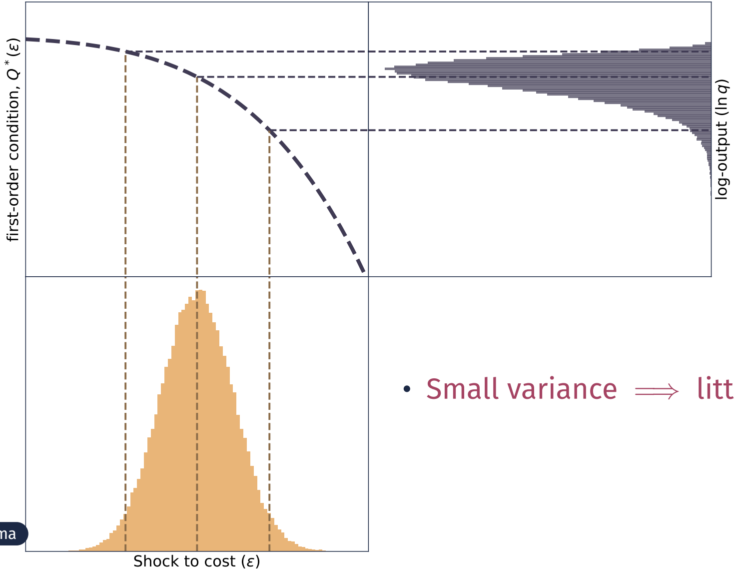
Variance and Skewness

Recall: Left-skew increases in ϕ for s-concave g and symm. Y: $\frac{\partial}{\partial \phi}\text{skew}[g(\phi Y)] \leq 0$



- Small variance $\implies$ little left-skew

Go to transformation lemma

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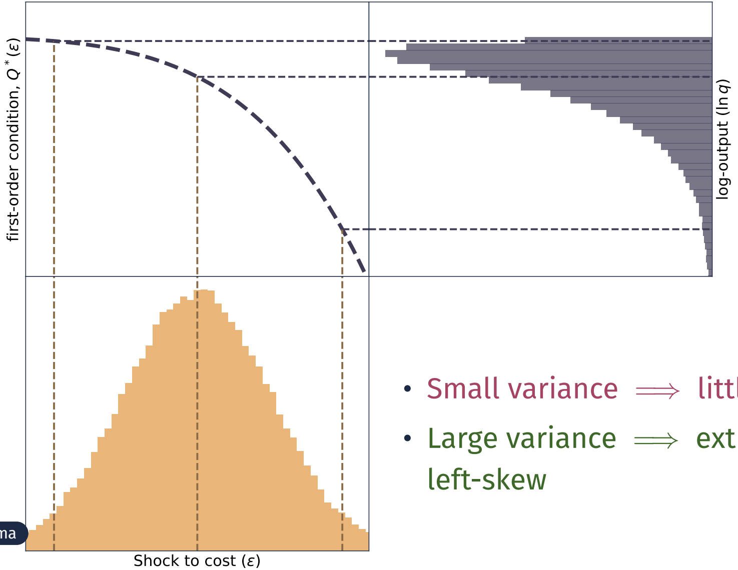

- Small variance $\implies$ little left-skew
- Large variance $\implies$ extreme left-skew

Go to transformation lemma

Business Cycle and Time Series Growth Rates

- **Larger shock variance $\implies$ more negative left-skew of $\hat{q}$**
- Suppose, the shock variance is countercyclical with

$$\sigma_t = \begin{cases} \sigma_{\text{rec}} \text{ for even } t \text{ (recessions)} \\ \sigma_{\text{boo}} \text{ for odd } t \text{ (boom)} \end{cases}$$

  with $\sigma_{\text{rec}} > \sigma_{\text{boo}}$.
- Then, for any $\alpha \in (0, 1]$, skewness of time-series growth rates is:

$$\text{skew}[\hat{q}_t - \hat{q}_{t-1}] = \begin{cases} \text{skew}[\hat{q}_{\text{recession}} - \hat{q}_{\text{boom}}] \text{ for even } t \\ \text{skew}[\hat{q}_{\text{boom}} - \hat{q}_{\text{recession}}] \text{ for odd } t \end{cases}$$

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slightly negatively skewed

strongly negatively skewed

Main Result: Market Power and Cyclicality of Growth-Skewness and Dispersion

Parametrize $\mathrm{skew}[\hat{q}_{\alpha,t} - \hat{q}_{\alpha,t-1}]$, the skewness of time-series growth rates, by market power, $\alpha \in (0,1]$. Suppose MSLD, IPT and counter-cyclical dispersion $(\ldots, \sigma_{\text{boo}}, \sigma_{\text{rec}}, \sigma_{\text{boo}}, \ldots)$ hold.

Then, the **time-series of growth-rate skewness** indexes for $\hat{q}_{\alpha,t} - \hat{q}_{\alpha,t-1}$ is **alternating pro-cyclically**: $(\ldots, \mathrm{skew}[\hat{q}_{\alpha,\mathrm{boo}} - \hat{q}_{\alpha,\mathrm{rec}}], \mathrm{skew}[\hat{q}_{\alpha,\mathrm{rec}} - \hat{q}_{\alpha,\mathrm{boo}}], \ldots)$ with

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Additionally, the **amplitude of the skewness sequence is strictly increasing in market power**:

$$\frac{\partial}{\partial \alpha}\left|\mathrm{skew}[\hat{q}_{\alpha,t} - \hat{q}_{\alpha,t-1}]\right| > 0.$$

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Theory Take-Away

1. Assume that size $\equiv$ market power.
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Simulation Exercise

Can we use the Theory to Simulate Skewness?

- Theory is clear, but pertains to a **stylized environment**
  - Real world is messy: persistent processes, jumps, etc...
  - Shock variances do not strictly oscillate!
- But **theory delivers a recipe** how to simulate the stylized facts:
  - Stick in some more realistic process for u_t (use an AR(2) with jumps)
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  - Use two concave, decreasing mappings to model $Q^\star(\,\cdot\,;\alpha)$, $\alpha \in \{\alpha_{low}, \alpha_{hi}\}$
- Try it! Play with free parameters to roughly match scale of skewness index and mean over time

Go to simulation results

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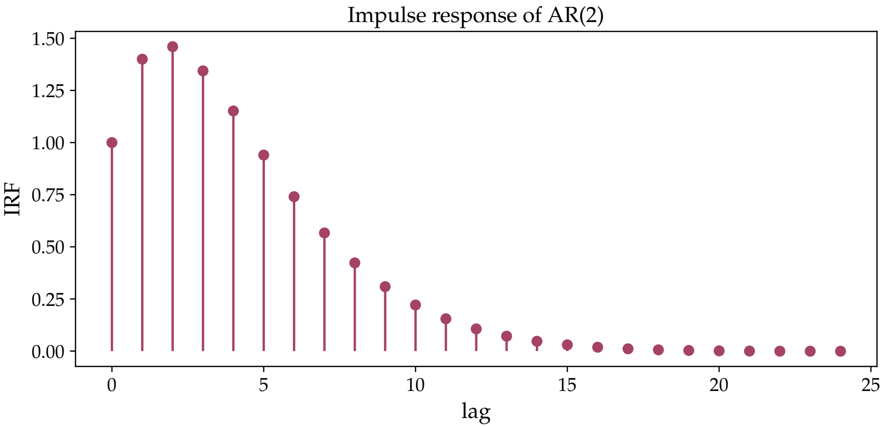
An AR(2) with Pareto jumps for u_t



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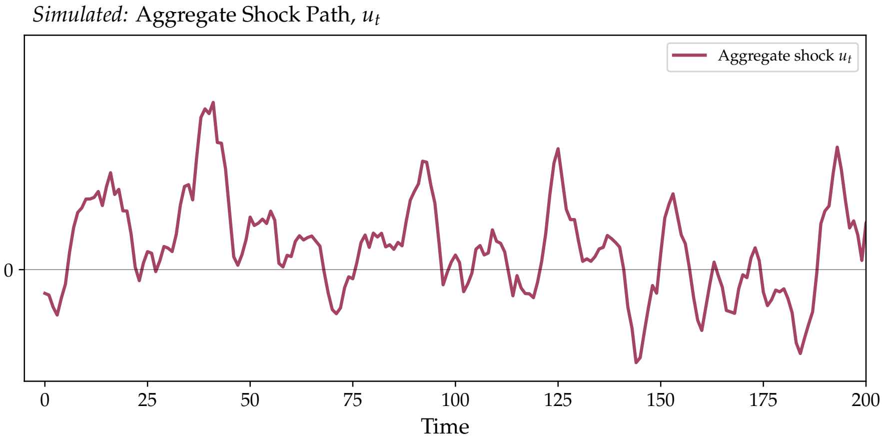
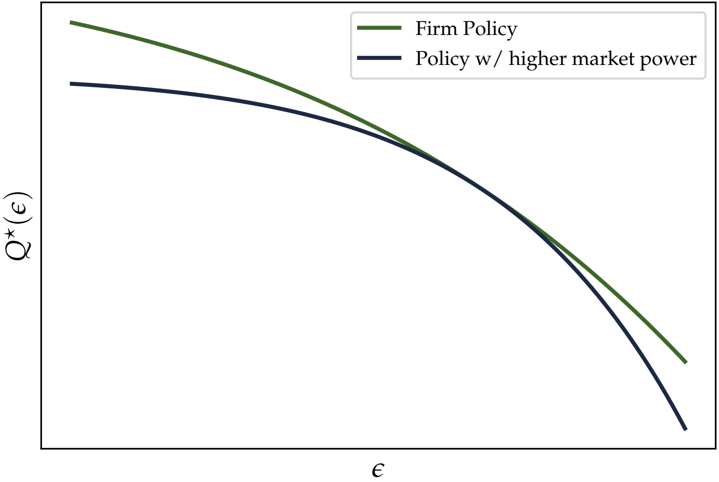
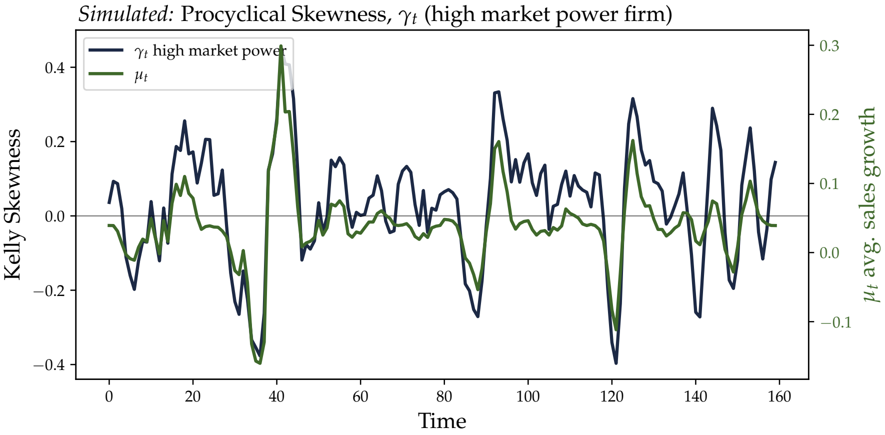


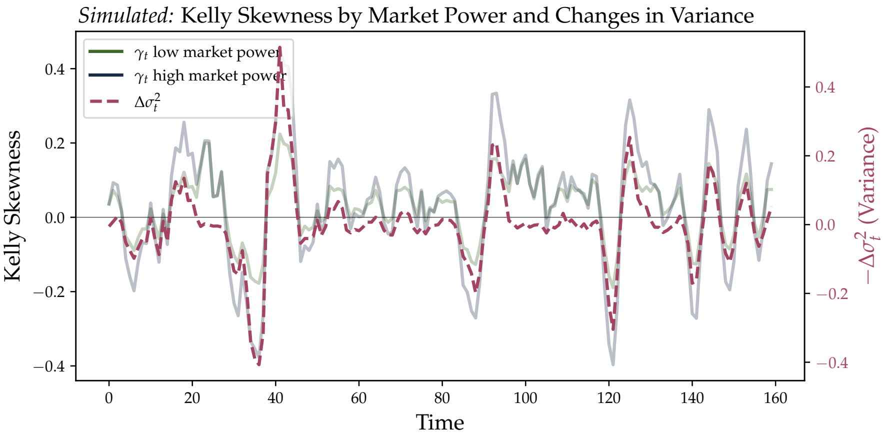
Concave, decreasing mappings for $Q^{\star}(\,\cdot\,;\alpha)$, $\alpha \in \{\alpha_{low}, \alpha_{hi}\}$



Simulated co-movement between μ_t and γ_t



Simulated co-movement between γ_t and $\Delta\sigma_t^2$ — for each α!



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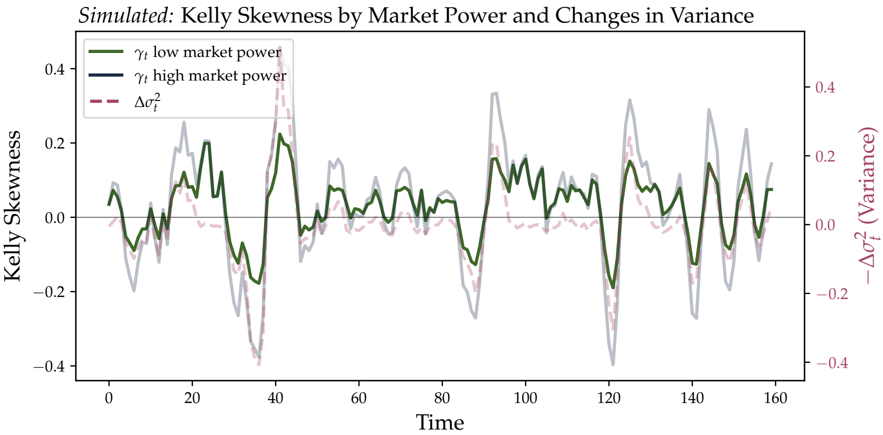

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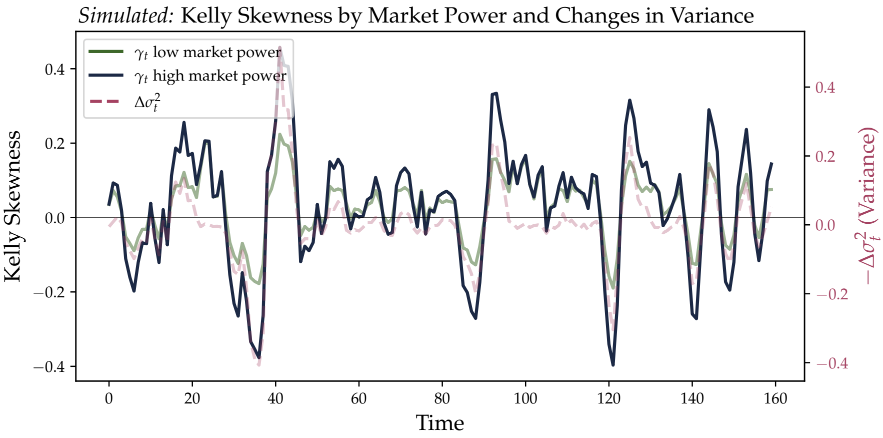

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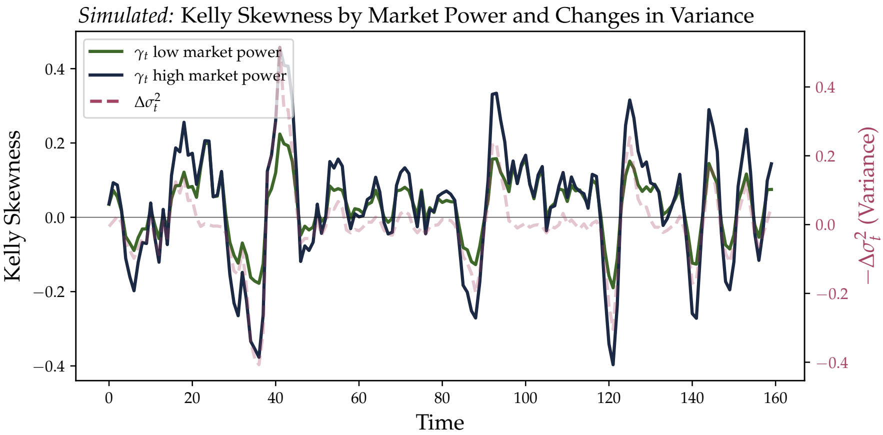

Empirics

The theoretical model implies following empirical hypotheses:

H0: Cross-sectional changes in output variance are a strong predictor of cross-sectional skewness, and more so for large firms. Skewness is pro-cyclical, and especially for large firms. Variance is counter-cyclical.

H1: Aggregate shocks ($u_t \neq 0$) cause **aggregate dip in skewness of growth rates**. The dip is **more pronounced for largest firms**.

H2: In a decomposition of growth rates into aggregate and idiosyncratic shocks, skewness of **aggregate shock explains skewness of growth rates** well.

These are the stylized facts.

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We confirm this by estimating impulse-response functions to a battery of off-the-shelf shocks.

▸ Go to analysis

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In a PCA, an aggregate component explains about 75% of skewness in sales growth. Go to analysis

H3: Split firms in high-(absolute-)exposure and low-exposure samples. **Skewness of high-exposure** firms **reacts more strongly** than that of low-exposure firms in response **to an exogenous aggregate shock**.

H4: Slice the firm sample by industry (not size). Then there is a strong **positive relationship** between the time-series **variance of the skewness index** and the **average HHI of the industry**.

H5: The stylized **facts hold in a disjoint sample** of listed European firms.

We confirm this using risk factors and exposures to COVID shocks from Davis et al. (2025). Go to analysis

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Examine and confirm using NAICS classifications. ▸ *Go to analysis*

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We reproduce cyclicality and size related stylized facts in Compustat Global.

Go to analysis

Conclusion

- **Theoretical insight:** aggregate shocks are all you need! Can generate cross-sectional moments of heterogeneous growth:
    - aggregate shock + heterogeneous exposures $\Rightarrow$ countercyclical variance
    - countercyclical variance + market power $\Rightarrow$ procyclical, monotone skewness
- **Empirical insight:** new pattern of business cycle statistics (monotone skewness)
    - Patterns are in line with **market power explanation**
    - Cross-sectional left-skewness can be created by aggregate 'disaster' shocks
    - *But different skewness in two cross-sections* $\not\Rightarrow$ *two different aggregate shocks or even different exposures!*

$\Rightarrow$ Skewness of realized growth distributions can be driven by **shock exposure or market power**

$\Rightarrow$ Also: concave policy of monopolist implies disproportionate reactions to negative shocks (be careful when using growth as a metric to hand out subsidies during crises!)

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Thank you!

References

Amiti, M., Itskhoki, O., and Konings, J. (2019). International shocks, variable markups, and domestic prices. *The Review of Economic Studies*, 86(6):2356–2402.

Autor, D., Dorn, D., Katz, L. F., Patterson, C., and Van Reenen, J. (2020). The fall of the labor share and the rise of superstar firms*. *The Quarterly Journal of Economics*, 135(2):645–709.

Baqaee, D. R., Farhi, E., and Sangani, K. (2024). The darwinian returns to scale. *Review of Economic Studies*, 91(3):1373–1405.

Berman, N., Martin, P., and Mayer, T. (2012). How do different exporters react to exchange rate changes? *The Quarterly Journal of Economics*, 127(1):437–492.

Bloom, N. (2009). The impact of uncertainty shocks. *econometrica*, 77(3):623–685.

Davis, S. J., Hansen, S., and Seminario-Amez, C. (2025). Macro shocks and firm-level response heterogeneity. Technical report, National Bureau of Economic Research.

De Loecker, J. and Warzynski, F. (2012). Markups and firm-level export status. *American Economic Review*, 102(6):2437–71.

Guvenen, F., Ozkan, S., and Song, J. (2014). The nature of countercyclical income risk. *Journal of Political Economy*, 122(3):621–660.

Higson, C., Holly, S., and Kattuman, P. (2002). The cross-sectional dynamics of the us business cycle: 1950–1999. *Journal of Economic Dynamics and Control*, 26(9-10):1539–1555.

Higson, C., Holly, S., Kattuman, P., and Platis, S. (2004). The business cycle, macroeconomic shocks and the cross-section: the growth of uk quoted companies. *Economica*, 71(282):299–318.

Ilut, C., Kehrig, M., and Schneider, M. (2018). Slow to hire, quick to fire: Employment dynamics with asymmetric responses to news. *Journal of Political Economy*, 126(5):2011–2071.

Kimball, M. S. (1995). The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit & Banking*, 27(4):1241–1290.

Krugman, P. R. (1979). Increasing returns, monopolistic competition, and international trade. *Journal of international Economics*, 9(4):469–479.

Lerner, A. (1934). The concept of monopoly and the measurement of monopoly power. *The Review of Economic Studies*, 1(3):157–175.

Matsuyama, K. and Ushchev, P. (2022). Selection and sorting of heterogeneous firms through competitive pressures. CEPR Discussion Papers 17092, C.E.P.R. Discussion Papers.

Melitz, M. J. (2018). Competitive effects of trade: theory and measurement. *Review of World Economics*, 154:1–13.

Palmer, J. A. (2003). Relative convexity. *ECE Dept., UCSD, Tech. Rep*.

Salgado, S., Guvenen, F., and Bloom, N. (2025). Skewed business cycles. *Working paper*.

Appendix

Appendix: Theory

What does "more concave" mean?

Formal definition due to Palmer (2003):

Definition: Relative Concavity

Consider two strictly monotone functions f and g. f **is concave relative to** g if there exists a strictly increasing, strictly concave function s such that $f = s \circ g$. We write $f \prec g$.

▸ Go to concavity-figure

What do relative concavity and RV-variance imply for skewness?

Lemma: Skewness of Transformed RVs

Let Y be a random variable, continuously and symmetrically distributed with $\mathbb{E}[Y] < \infty$. Let $\phi > 0$ be constant, and $g(\cdot)$ be a concave and increasing function over the support of Y (resp. ϕY). Then:

Go to concavity figure Go to variance figure

Skewness of Transformed Random Variables

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$$\text{skewed } \hat{q} \iff \text{concave } Q^{\star}$$

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$$\text{more left-skew} \iff \text{more concave } Q^{\star} \iff \text{more market power}$$

Go to concavity figure Go to variance figure

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$$\frac{\partial}{\partial \phi} \mathrm{skew}[g(\phi Y)] < 0,$$

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Go to concavity figure   Go to variance figure   variance affects skewness! (later)

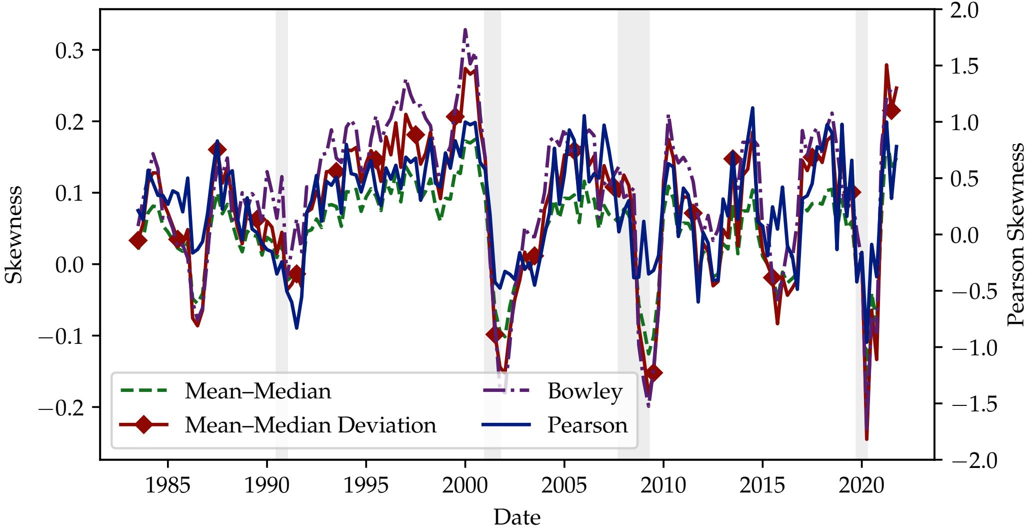
Theor. Properties vs. Empirical Literature

- MSLD is prevalent e.g. in trade literature (Krugman (1979)), popular aggregators satisfy MSLD (Kimball, 1995) (CES does not!), recent attention in e.g. Matsuyama and Ushchev (2022)
- There is empirical support for MSLD and IPT (Berman et al., 2012; Baqaee et al., 2024; Amiti et al., 2019)
- There is also strong empirical support that larger firms have more market power (De Loecker and Warzynski, 2012; Autor et al., 2020)
- Evidence that input shock variance is countercyclical: Bloom (2009); Davis et al. (2025) plus previously cited.

▸ Back to theory recap

Appendix: Stylized Facts

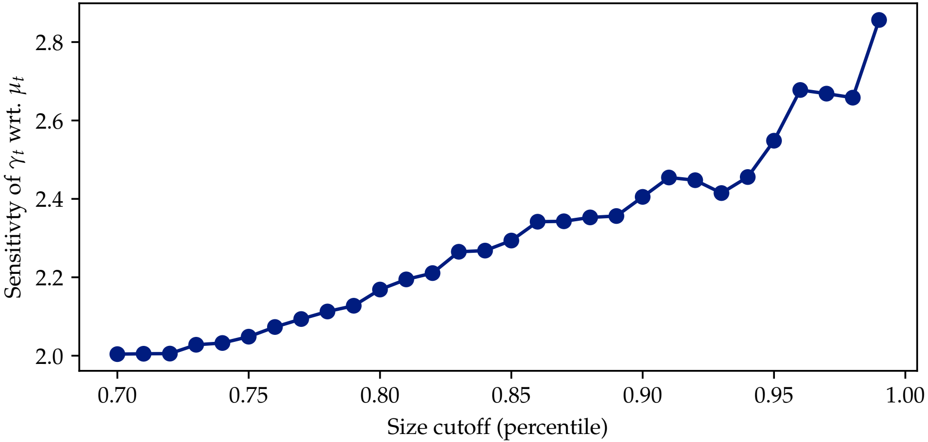
Robustness: Skewness measures



▸ Back to s-facts

Robustness: Procyclical skewness for increasing size cutoffs

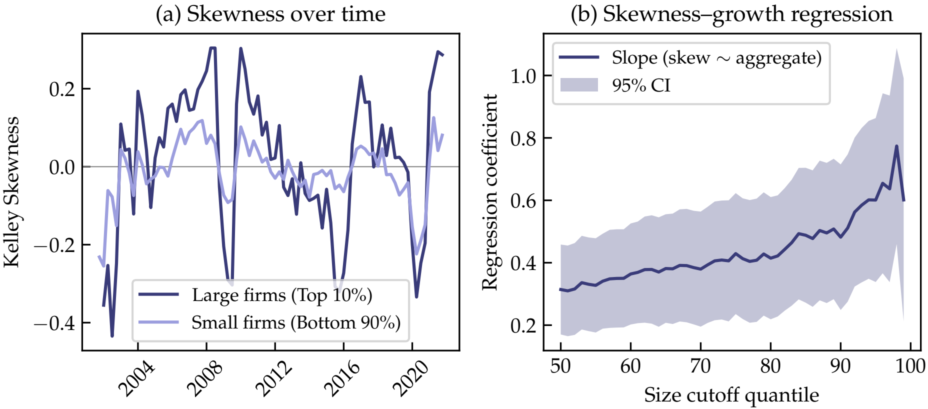
$$\gamma_{t,p} = \alpha_p + \beta_p \mu_t + \epsilon_t \tag{1}$$



Back to s-facts

Stylized Facts Compustat (Compustat Global)

Figure 4: Size-dependent skewness (ex. US)

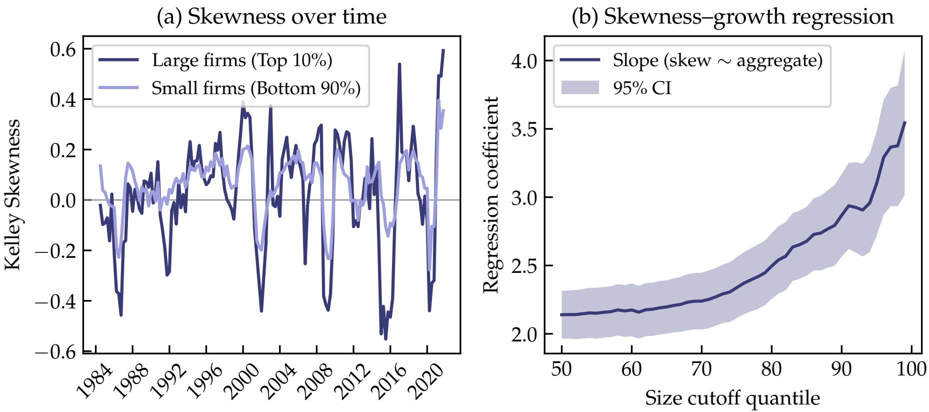


Note: Size groups are defined based on average real sales over previous three years. The sample is *Compustat Global* which excludes US-listed firms.

▸ Back to s-facts ▸ Back to hypotheses

Stylized Facts Compustat (Paper Figure)

Figure 5: Size-dependent skewness (U.S. only)



Note: Size groups are defined based on average real sales over previous three years. The standard deviation of Kelley skewness for large firms is about 0.23 — more than twice the corresponding value of 0.11 for small firms.

▸ Back to s-facts ▸ Back to hypotheses

Appendix: Data Description

Compustat Data

Dataset Overview:

- Compustat: US public firms, quarterly frequency, over 35 years.
- All firms are large by global standards (avg. assets of USD 2.8bn)
- Key variable: Real sales $s_{i,t}$; growth defined as $g_{i,t} = \ln(s_{i,t}) - \ln(s_{i,t-4})$.
- Aggregate sales growth (size-weighted):

$$g_t = \frac{\sum_i g_{i,t} s_{i,t-4}}{\sum_i s_{i,t-4}}$$

Firm Size Characteristics:

- Firms in Compustat sample are large relative to the universe of US firms.
- Largest 10% of firms account for approximately 70% of total sales.
- Top 30% of firms represent over 90% of total sales.

Go to IRFs Go to PCA

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▸ Go to IRFs ▸ Go to PCA

Appendix: Impulse Response Functions (H1)

Impulse Response Findings

Aggregate shocks studied:

- Monetary, oil supply, credit, uncertainty, sentiment, TFP shocks.

Key results:

- All shocks induce significant declines in skewness (0.02–0.06 points).
- Strong correlation between impulse responses of skewness and aggregate sales growth (0.89-0.98).
- Large firms exhibit more pronounced skewness response than smaller firms.

▸ data description

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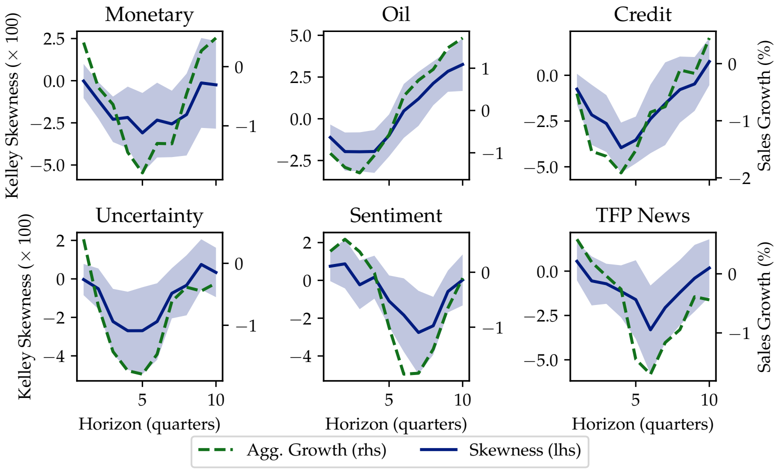
▸ data description

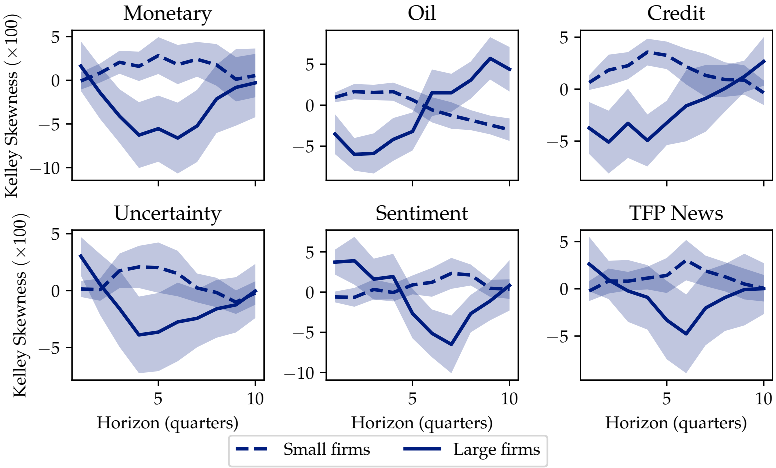
Skewness and average sales growth response

- **Five** out of six shocks induce negative skewness, **all** induce co-movement
- **Takeaway:** Co-movement of μ_t and γ_t holds for **structural** shocks



Skewness IRFs according to size groups

- **Five** out of six shocks induce more skewness for large firms
- **Takeaway:** Likely that skewness response a result from skewed responses of large firms



▸ Back to hypotheses

Appendix: Sales Growth Decomposition and Skewness (H2)

Aggregate vs. Idiosyncratic Components

Decomposition approach via PCA:

$$g_{i,t} = \delta_i + a_{i,t} + u_{i,t}, \quad a_{i,t} = \beta_i' F_t, \quad g_{i,t} \equiv \Delta \log q_{i,t},$$

where β_i are estimated factor loadings and F_t is an aggregate factor.

Results:

- Aggregate component ($a_{i,t}$) strongly correlated with skewness in sales growth.
- Aggregate component explains 75% of skewness variation; idiosyncratic component (δ_i) is less significant (˜25%).

▸ data description

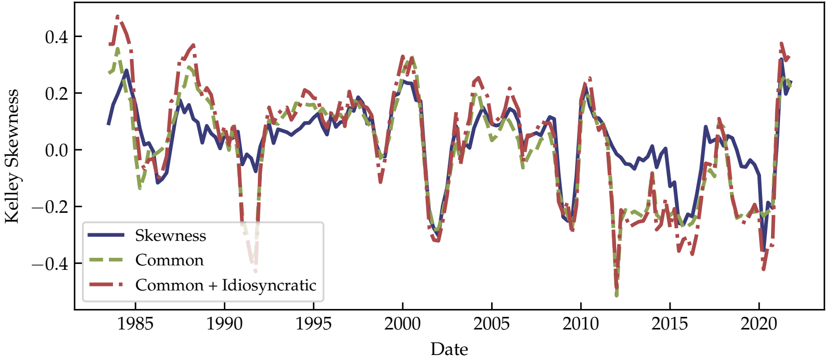
Factor Decomposition Results

Key observations from PCA:

- Single aggregate factor accounts for 79% skewness variation.
- Aggregate factors explain relatively little (30%) of individual firm-level variation.
- Thus, skewness is driven by heterogeneous firm-level responses to common shocks rather than purely idiosyncratic variation.

Decomposition of Skewness

- Skewness in the *idiosyncratic* component adds little beyond the procyclical pattern in the *common* component.
- The sum of common and idiosyncratic contributions (green) closely matches the overall skewness measure (blue)



▸ Back to hypotheses

Appendix: Risk Factors (H3)

COVID risk exposures and skewness Davis et al. (2025)

- Risk exposures λ_i to COVID shock be Davis et al. (2025)
- Split sample: $i \in \mathcal{H}$ if $|\lambda_i|$ is above median **absolute** exposure, $|\lambda_i|_{0.5}$. Otherwise, $i \in \mathcal{L}$
- **Prediction:** growth rate skewness of $\gamma_t(\mathcal{H}) < \gamma_t(\mathcal{L})$ on impact, $\gamma_t(\mathcal{H}) > \gamma_t(\mathcal{L})$ during recovery.

Back to hypotheses

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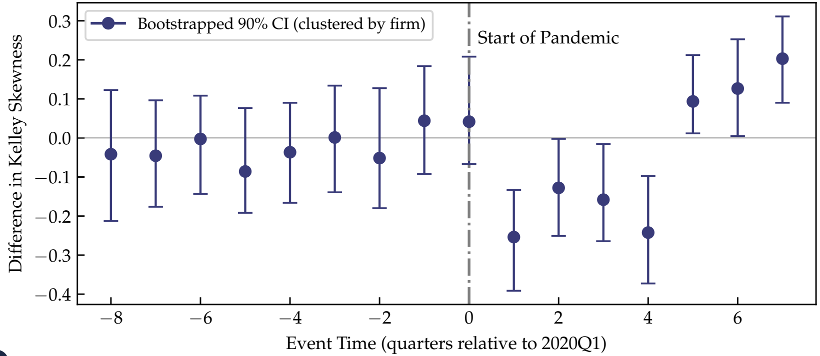
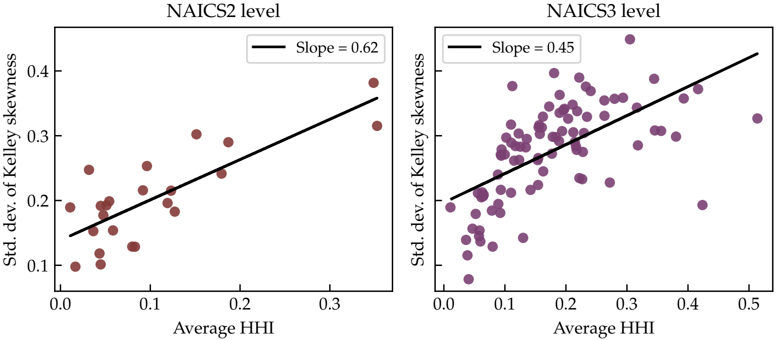


▸ Back to hypotheses

Appendix: Concentration drives Skewness (H4)

Skewness and Concentration

- If market power is the driving force, then skewness within sectors with large HHI should have larger amplitude, hence time-series variance
- Strongest correlation between amplitude of skewness fluctuations and HHI concentration for coarsest sectors
- Constraining firms to be more equal (finer sectors) mutes relationship (explanation: smaller cross-sections and measurement error)



▸ Back to hypotheses

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